Anode Spot Oscillations

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Summary

Experimental studies of alkali-vapor tubes having four identical spherical molybdenum anodes, each in its own cylindrical well, indicate that the periods of the oscillations observed near the anodes are closely related to the expected diffusion decay-times of bursts of ionization released from the anode surface.

Theoretical analysis of the steady state ion density, current density, and electric field near the anode has been completed for a one-dimensional configuration with equal concentrations of ions and electrons.

1. Introduction

Oscillations in tube voltage and emitted light have been observed to occur at the anode in low-pressure cesium (or other alkali metal) vapor plasma tubes. These spontaneous and reproducible oscillations are similar in behavior to those found in cesium thermionic diodes and sometimes regarded as possible sources of a.c. power directly from heat. Experimental studies of these oscillations are much more convenient in transparent, relatively low-temperature plasma tubes than in the thermionic diodes per se, because optical observations are possible and other important parameters such as anode temperature and cesium vapor pressure are more easily controlled independently.

During this report period, experiments have been mostly with 35 mm. i.d. tubes, about 30 cm long, containing one or four spherical molybdenum anodes. Cesium, rubidium, and potassium vapors have been used in an attempt to relate anode spot properties to the properties of the vapor.

2. Experiments with Four-anode Tubes

We have already extablished that the anode spot oscillations are associated with bursts of ionization, at or very near the anode surface, followed by a rapid fall in the local electric field strength, after which the excess ion and electron density dissipates by diffusion and conduction while the electric field intensity builds up to a value sufficient to cause another burst of ionization and repetition of the cycle. The theory is intrinsically non-linear, and solutions to the basic equations describing the time-dependent behavior of the field are therefore not easy to solve. Physically, however, one expects that the period of the oscillations will be approximately equal to the decay time of the ionization burst, and that the decay time should be an expression like $\tau = \lambda^2/D_a$, where λ is the diffusion length and D_a is the ambipolar diffusivity. (If conduction, rather than diffusion, is the dominant mechanism for removal of the ions, a somewhat different expression is found, but in any case a length is involved)

Figure 1 shows a typical plot of the oscillation frequency as a function of tube current for constant pressure.

This particular curve, for a spherical molybdenum anode of 2.06 mm diameter in its anode tube of 6.15 mm i.d., was obtained with a four-anode tube containing rubidium vapor. The notable point is that for sufficiently high currents, the oscillation frequency becomes almost independent of current, and it is this asymptotic frequency that is expected to be related to the diffusion time. Since the electric field strength is higher at lower currents, conduction aids diffusion in the removal of ions and electrons, and the frequency is therefore expected to be higher than that resulting from simple diffusive loss.

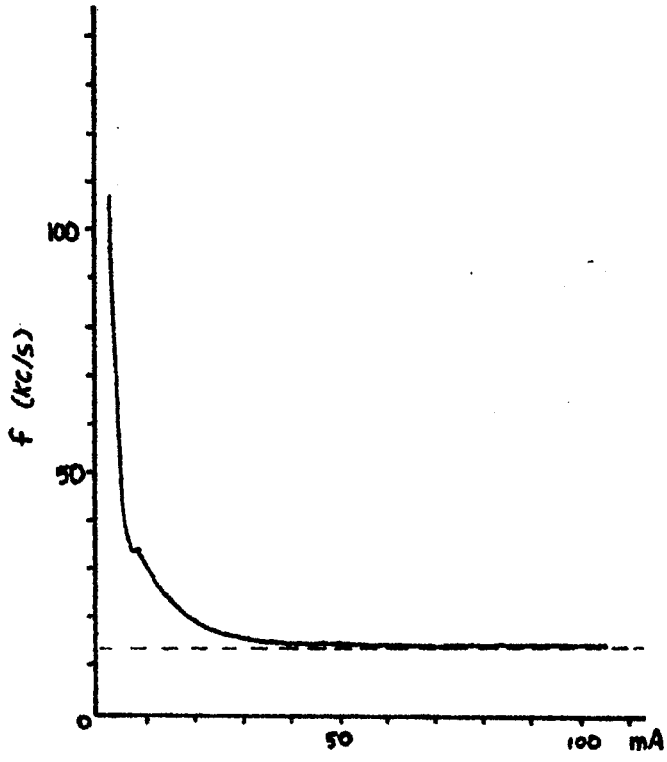

Fig. 1. Frequency of oscillation vs. current for anode No. 4 of tube 123, rubidium, with reservoir at 170°C and plasma at 235°C.

Dimensionally, the asymptotic frequency should go as D_a/λ^2, where λ is the characteristic diffusion length. Although λ should depend entirely on the geometry of the electrode in its surrounding tube, an exact calculation of its value has not been made. Instead, a plot of the asymptotic frequency as a function of $1/r^2$ is shown in Fig. 2 for three values of rubidium pressure. The four values of the tube radius r are, respectively, 0.308, 0.228, 0.165, and 0.110 cm, and the spherical anodes are all 0.206 ± 0.003 cm diameter. The limiting frequencies are, for each pressure, monotone increasing functions of $1/r^2$, and at least three points per curve lie on a straight line. No explanation is yet available for the additional delay that causes the three lines to extrapolate to $1/r^2 = 10\ cm^{-2}$ at $f = 0$.

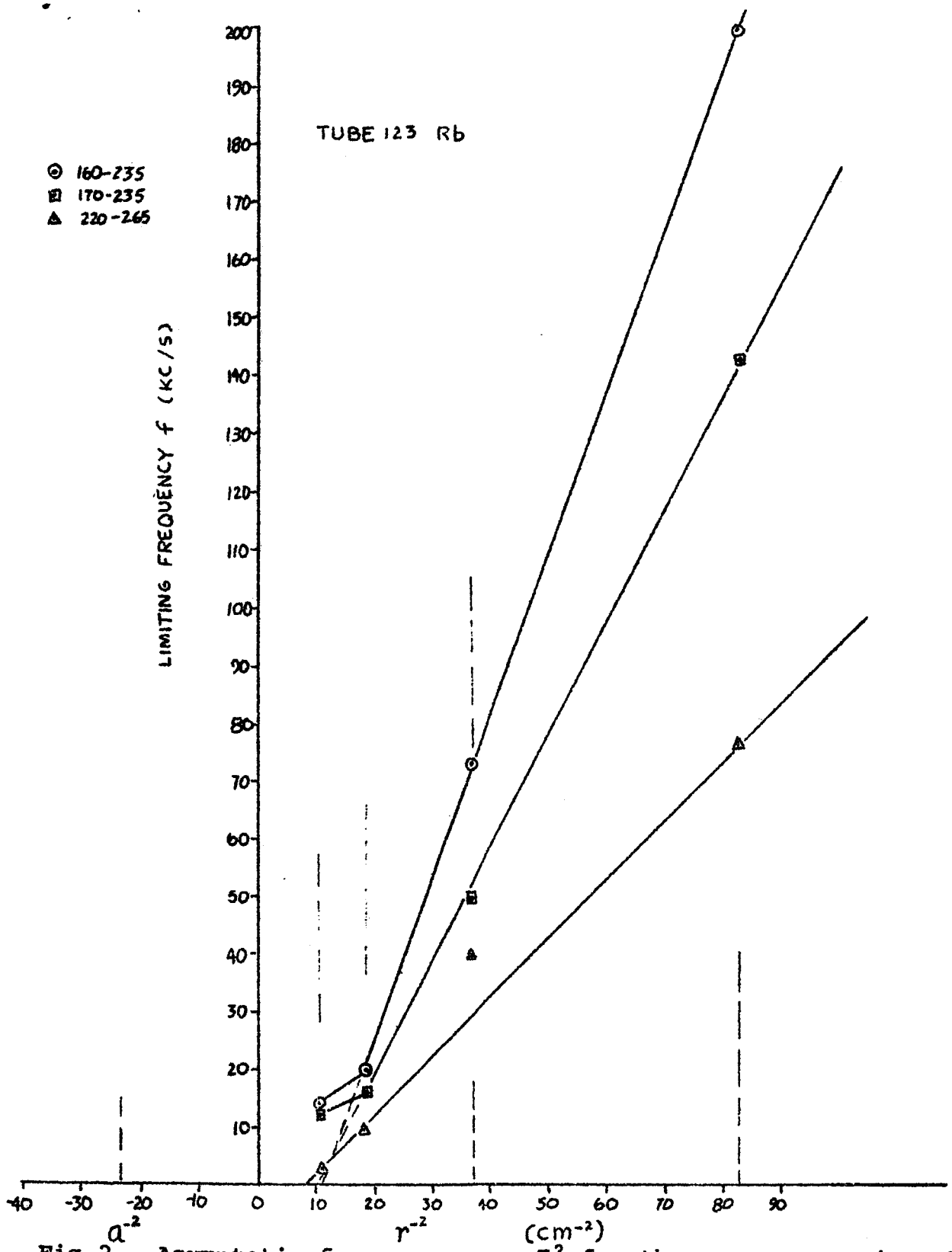

Fig.2. Asymptotic frequency vs. r^{-2} for three pressures in rubidium. The four tube radii are indicated by dashed vertical lines, and the line at $-a^{-2}$ corresponds to the anode diameter. The numbers in the upper left indicate respectively the reservoir and tube temperatures in °C of the corresponding graph points.

As the pressure, and the thus neutral density, increases, the diffusivity decreases, and the frequency would therefore be expected to decrease. Although the correct density calculations have not yet been made for rubidium vapor, it is clear that the vapor density increases with reservoir temperature and that the frequency dependence upon pressure is therefore at least qualitatively correct. A more careful study of this kind will be made when time permits.

Similar experiments in the cesium vapor tube, for which the density-temperature relationship is well-known, were unsuccessful in that there were usually no oscillations for the two smallest anode-tubes, and the other two were subject to mode changes and multiple spot formation. A repetition of the experiments with this tube will be made in order to check the data for single-spot oscillations.

The tentative experimental conclusions so far are that the expected diffusion decay-time of the ionization bursts accounts qualitatively for the dependence of the observed oscillation period on tube radius and pressure.

3. Other Experiments

A tube with a single spherical anode in cesium vapor has been studied in detail, in order to determine its modes of operation as functions of temperature and current. The transitions from m spots to n spots seem to follow smooth curves in temperature-current space, but these are not yet understood.

In another experiment, the anode was made of nickel ribbon, just like the cathode. Anode spot oscillations were observed with the anode at the ambient temperature of the tube, and no significant change in the oscillation frequency or number of spots was observed when the anode was heated to about 1000°C by passing 12 amperes through it. A more sensitive test, which has not yet been done, is the examination of the possible shifts in mode transition points as a function of anode temperature. A more suitable anode structure is being built for this purpose.

4. Theoretical Results

A calculation has been made of the steady state behavior of the ion and electron cloud near the anode, assuming charge neutrality in a one-dimensional geometry. The basic equations to be solved are:

$$(1) \qquad \Gamma_e = - D_e \frac{\partial n}{\partial x} - \mu_e E N ,$$

$$(2) \qquad \Gamma_p = - D_p \frac{\partial n}{\partial x} + \mu_p E N ,$$

$$(3) \qquad \frac{\partial \Gamma_e}{\partial x} = R = \frac{\partial \Gamma_p}{\partial x} ,$$

where Γ_e and Γ_p are the electron and ion particle current densities, E is the electric field, N is the electron and ion concentration, D and μ are the diffusivity and mobility, and R is the effective net ionization rate. The first two equations define the particle currents and the third is just the equation of continuity for time-independent one-dimensional flow, with the production term necessarily equal for ions and electrons.

The key to finding analytic solutions to these equations lies in the representation of the term R. No exact analytic expression for R exists, but the net ionization rate must reflect dependence on electron density, electric field intensity, and the physically important radial loss that occurs in laboratory plasmas but not in one-dimensional models of them. If the reasonable choice is made that the net ionization rate is $R = \alpha\mu_e N(E-E_0)$, where $\alpha\mu_e$ is a proportionality constant and E_0 is the value of the electric field in the plasma, well away from the anode, then the net ionization rate is zero when $E = E_0$, positive when $E > E_0$, and negative when $E < E_0$, but always proportional to the electron density N. A negative net ionization rate simply means that the radial loss rate removes ions and electrons faster than the true ionization rate replaces them. This particular choice of R, however, makes the basic equations quite intractable, so a search was made for a nearly equivalent form of R that was easier to handle.

In the plasma, where there are no axial concentration gradients, $\Gamma_e = - \mu_e E N$, so a neutral choice is to let $R = -\alpha(\Gamma_e - \Gamma_{e0})$, where Γ_{e0} is the electron current density in the plasma. This choice of R allows net ionization to depend upon diffusion current, and for this reason it may not always be physically valid, but it does allow a steady state solution to be made. Equation (3) integrates immediately to give

$$(4) \qquad \Gamma_e = - N_0 E_0 (\mu_e + \mu_p e^{-\alpha x}) ,$$

where the boundary conditions chosen are that the total current density $j_0 = e(\Gamma_p - \Gamma_e)$ is constant and that the ion current

density Γ_p is zero at x=0, the location of the anode.

Further solution of the equations yields

$$(5) \qquad N(x) = N_0\left(1 - \frac{E_0\mu_p}{\alpha D_a} e^{-\alpha x}\right) ,$$

where the ambipolar diffusivity

$$D_a = \frac{D_e\mu_p + D_p\mu_e}{\mu_e + \mu_p} , \quad \text{and, finally}$$

$$(6) \qquad E(x) = \frac{E_0\left[1+(\mu_p/\mu_e)(1-D_e/D_a)e^{-\alpha x}\right]}{1 - (E_0\mu_p/\alpha D_a)e^{-\alpha x}} .$$

These solutions all yield the plasma values of the variables as $x \to \infty$, as required, and they describe the variables at the anode when x=0.

An analysis of the physical behavior of the steady-state system, too lengthy to be included here, yields that the proper choice of α is approximately $\alpha \simeq E_0/V_e$, where V_e is the electron temperature in volts. Typical values for cesium plasmas are that $E_0 \simeq 1$ V/cm and $V_e \simeq 0.2$ eV, giving a typical value of α as 5/cm, or a decay length of about 2 mm. This is a typical visual dimension of the anode spot phenomenon.

For this choice of α, the electric field is at least as large at the anode as in the plasma.

It is further conjectured, although the analysis is not complete, that the plasma is stable to perturbations in ion density, and that the decay time-constant of the perturbations is approximately Λ^2/D_a, where Λ is the diffusion length described previously.

A three-dimensional calculation is necessary for the correct evaluation of this conjecture, but further analysis of the one-dimensional analytically soluable problem will be carried out before a serious study of the three-dimensional problem is begun.

Further results of this study will be included in the next semiannual progress report, and a more detailed report on the present studies is being prepared for issue as a University of Miami Plasma Physics Bulletin.